Unit 3
Logarithms

EXERCISE 3.1

Q1. Express each of the following numbers in scientific notation.

(i)	5700	(ii)	49,800,000
(iii)	96,000,000	(iv)	416.9
(v)	83,000	(vi)	0.00643
(vii)	0.0074	(viii)	60,000,000
(ix)	0.00000000395	(x)	$\frac{275,000}{0,0025}$

Note:

A number written in the form $a \times 10^n$, where $1 \leq a \leq 10$ and n is an integer, is called the scientific notation.

Solution:

(i) 5700

$\frac{5700}{1000} \times 1000 = 5.7 \times 10^3$

(ii) 498 00 000

$\frac{49800000}{10000000} \times 10000000 = 4.98 \times 10^7$

(iii) 96 000 000

$\frac{96000000}{10000000} \times 10000000 = 9.6 \times 10^7$

(iv) 416.9

$\frac{4169}{10} = 4169 \times 10^{-1}$

$\frac{4169}{1000} \times 1000 \times 10^{-1} = 4.169 \times 10^{3-1} = 4.169 \times 10^2$

(v) 83 000

$\frac{83000}{10000} \times 10000 = 8.3 \times 10^4$

(vi) 0.00643

$\frac{00643}{100000} = 643 \times 10^{-5}$

$\frac{643}{100} \times 100 \times 10^{-5} = 6.43 \times 10^{2-5} = 6.43 \times 10^{-3}$

(vii) 0.0074

$\frac{0074}{10000} = 74 \times 10^{-4}$

$\frac{74}{10} \times 10 \times 10^{-4} = 7.4 \times 10^{1-4} = 7.4 \times 10^{-3}$

(viii) 60 000 000

$\frac{60000000}{10000000} \times 10000000 = 6.0 \times 10^{7}$

(ix) 0.000 000 00 395

$\frac{00000000395}{100000000000} = 395 \times 10^{-11}$

$\frac{395}{100} \times 100 \times 10^{-11} = 3.95 \times 10^{2-11} = 3.95 \times 10^{-9}$

(x) $\frac{275000}{0.0025} = \frac{275 \times 10^{3}}{25 \times 10^{-4}}$

$= 11 \times 10^{3+4}$

$= \frac{11}{10} \times 10 \times 10^{7} = 1.1 \times 10^{7+1}$

$= 1.1 \times 10^{8}$

Q2. Express the following numbers in ordinary notation.

(i) 6×10^{-4} **(ii)** 5.06×10^{10}

(iii) 9.018×10^{-6} **(iv)** 7.865×10^{8}

Solution:

(i) 6×10^{-4}

$= 6.0 \times 10^{-4} = \frac{6}{10^{4}}$

$= \frac{6}{10000} = 0.0006$

(ii) 5.06×10^{10}

$= \frac{506}{100} \times 10^{10} = 506 \times 10^{10-2}$

$= 506 \times 10^{8} = 50,600,000,000$

(iii) 9.018×10^{-6}

$= \frac{9018}{1000} \times 10^{-6} = 9018 \times 10^{-6-3}$

$= 9018 \times 10^{-9} = \frac{9018}{1000000000}$

$= 0.000009018$

(iv) 7.865×10^{8}

$= \frac{7865}{1000} \times 10^{8} = 7865 \times 10^{8-3}$

$= 7865 \times 10^{5} = 786,500,000$

EXERCISE 3.2

Q1. Find the common logarithms of the following numbers.

(i) 232.92 **(ii) 29.326**

(iii) 0.00032 **(iv) 0.3206**

Solution:

(i) 232.92

232.92 can be rounded off as 232.9. The characteristic is 2 as there are 3 digits.

To find mantissa we follow the row of 23 and reach the column of 2 to get 3655. In the same row in the difference column of 9 we see 17. Add 3655 and 17 and get mantissa .3672.

So log 232.92 = 2.3672

(ii) 29.326

29.326 can be rounded off as 29.33. The characteristic is 1 as there are 2 digits.

To find mantissa we follow the row of 29 and reach the column of 3 to get 4669. In the same row in the difference column of 3 we see 4. Add 4669 and 4 to get mantissa .4673.

So log 29.326 = 1.4673

(iii) 0.000 32

The characteristic is -4 as which is written as $\bar{4}$.

To find mantissa we follow the row of 32 and reach the column of 0 to get 5051. So mantissa is 0.5051.

So log 0.000 32 = $\bar{4}.5051$

(iv) 0.3206

The characteristic is -1 as which is written as $\bar{1}$.

To find mantissa we follow the row of 32 and reach the column of 0 to get 5051. In the same row in the difference column of 6 we see 8. Add 5051 and 8 to get mantissa 0.5059.

So log 0.3206 = $\bar{1}.5059$

Q2. If log 31.[illegible] = 1.4926, find values of following

(i) log [illegible] **(ii) log 310.9**

(iii) log [illegible]

(iv) log [illegible] **using tables.**

Solution:

Given log 31.09 = 1.4926

(i) In log 3.109 the characteristic is 0 and mantissa is 0.4926

So log 3.109 = 0.4926

(ii) In log 310.9 the characteristic is 2 and mantissa is 0.4926

So log 310.9 = 2.4926

(iii) In log 0.00 3109 the characteristic is $\bar{3}$ and mantissa is 0.4926

So log 0.003109 = $\bar{3}$.4926

(iv) In log 0.3109 the characteristic is $\bar{1}$ and mantissa is 0.4926

So log 0.3109 = $\bar{1}$.4926

Q3. Find the numbers whose common logarithms are

(i) 3.5621 **(ii) 1.7427**

Solution:

(i) 3.4521

Reading along the row corresponding to .56 we get 3648 at the intersection of this row and column of 2. The number at the intersection of this row and the mean difference column of 1 is 1. Adding 3648 and 1 we get 3649.

Since the characteristic is 3, the number has four digits. So the required number is **3649.**

(ii) $\bar{1}$.7427

Reading along the row corresponding to .74 we get 5521 at the intersection of this row and column of 2. The number at the intersection of this row and the mean difference column of 7 is 9. Adding 5521 and 9 we get 5530.

Since the characteristics is $\bar{1}$. So the required number is **0.5530**.

Q4. What replacement for the unknown in each of following will make the statement true?

(i) $\log_3 81 = L$ **(ii)** $\log_a 6 = 0.5$

(iii) $\log_5 n = 2$ **(iv)** $10^p = 4$

Solution:

(i) $\log_3 81 = L$

$3^L = 8$

$3^L = 3^4$

$L = 4$

(ii) $\log_a 6 = 0.5$

$a^{0.5} = 6$

$a^{1/2} = 6$

$\sqrt{a} = 6$

By squaring on both sides

$a = 36$

(iii) $\log_5 n = 0.5$

$5^2 = n$

$n = 25$

(iv) $10^p = 4$

Taking log on both sides

$\log 10^p = \log 4$

or $p \log 10 = \log 4$

$p \times 1 = 0.6021$ $\quad (\because \log 10 = 1)$

$p = 0.6021$

Q5. Evaluate **(i)** $\log 2 \frac{1}{128}$

(ii) $\log 512$ **to the base** $2\sqrt{2}$

Solution:

(i) $\log 2 \frac{1}{128}$

Let $\log 2 \frac{1}{128} = x$

Exponential form is

$2^x = \frac{1}{128}$

$\therefore \quad 2^x = \frac{1}{2^7}$

or $2^x = 2^{-7}$

$x = -7$

(ii) $\log 512$ **to the base** $2\sqrt{2}$

Let $\log_{2\sqrt{2}} 512 = x$

Exponential form is

$(2\sqrt{2})^x = 512$

$\left(2 \times 2^{\frac{1}{2}}\right)^x = 2^9$

$(2^{3/2})^x = 2^9$ $\quad \left(1 + \frac{1}{2} = \frac{3}{2}\right)$

$2^{3x/2} = 2^9$

$$\frac{3x}{2} = 9$$

$$x = \frac{9 \times 2}{3} = \frac{18}{3} = 6$$

Q6. Find the value of x from the following statements.

(i) $\log_2 x = 5$ (ii) $\log_8 19 = x$

(iii) $\log_{64} 8 = \frac{x}{2}$ (iv) $\log_x 64 = 2$

(v) $\log_3 x = 4$

Solution:

(i) $\log_2 x = 5$

$2^5 = x$

So, $x = 32$

(ii) $\log_{81} 9 = x$

$(81)^x = 9$

$(9^2)^x = 9$

$9^{2x} = 9^1$

$2x = 1$

So, $x = \frac{1}{2}$

(iii) $\log_{64} 8 = \frac{x}{2}$

$(64)^{\frac{x}{2}} = 8$

$(8^2)^{\frac{x}{2}} = 8$

$8^x = 8^1$

So, $x = 1$

(iv) $\log_x 64 = 2$

$x^2 = 64$

$x^2 = (8)^2$

So, $x = 8$

(v) $\log_3 x = 4$

$3^4 = x$

$x = 3^4$

So, $x = 81$

EXERCISE 3.3

Q1. Write the following into sum or difference.

(i) $\log (A \times B)$ (ii) $\log \frac{15.2}{30.5}$

(iii) $\log \frac{21\times5}{8}$ (iv) $\log \sqrt[3]{\frac{7}{15}}$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$ (vi) $\log \frac{25\times47}{29}$

Solution:

(i) $\log (A \times B) = \log A + \log (B)$

(ii) $\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5$

(iii) $\log \frac{21 \times 5}{8} = \log 21 \times 5 - \log 8$

$= \log 21 + \log 5 - \log 8$

(iv) $\log \sqrt[3]{\frac{7}{15}} = \log\left(\frac{7}{15}\right)^{\frac{1}{3}}$

$= \frac{1}{3} \log \frac{7}{15}$

$= \frac{1}{3} [\log 7 - \log 15]$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log (22)^{\frac{1}{3}} + \log 5^3$

$= \frac{1}{3} \log 22 - 3 \log 5$

(vi) $\log \frac{25\times47}{29} = \log (25 \times 47) - \log 29$

$= \log 25 + \log 47 - \log 29$

Q2. Express $\log x - 2 \log x + 3 \log(x + 1) - \log(x^2 - 1)$ as a single logarithm.

Solution:

$\log x - 2 \log x + 3 \log (x + 1) - \log (x^2 - 1)$

$= (1-2)\log x + 3\log(x + 1) - \log(x + 1)(x - 1)$

$= -\log x + 3 \log (x + 1) - [\log (x + 1) + \log (x - 1)]$

$= -\log x + 3\log(x + 1) - \log(x + 1) - \log(x - 1)$

$= 2 \log (x + 1) - \log x - \log (x - 1)$

$= 2 \log (x + 1) - \log [\log x + \log (x - 1)]$

$= \log (x + 1)^2 - \log x(x - 1)$

$= \quad log \frac{(x+1)^2}{x(x-1)}$

Q3. Write the following in the form of a single logarithm.

(i) log 21 + log 5 (ii) log 25 – 2 log 3

(iii) 2 log x – 3 log y (iv) log 5 + log 6 – log 2

Solution:

(i) log 21 + log 5

= log 21 × 5

(ii) log 25 + 2 log 3

$= \quad \log 25 - \log 3^2 = \quad \log \frac{25}{3^2}$

(iii) 2 log x – 3 log y

$= \quad \log x^2 \times \log y^3 \quad = \quad \log \frac{x^2}{y^3}$

(iv) log 5 + log 6 – log 2

$= \quad \log 5 \times 6 - \log 2 \quad = \quad \log \frac{5 \times 6}{2}$

Q4. Calculate the following:

(i) $\log_3 2 \times \log_2 81$ (ii) $\log_5 3 \times \log_3 25$

Solution:

(i) $\log_3 2 \times \log_2 81$

$= \quad \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2} \quad = \quad \frac{\log 81}{\log 3}$

$= \quad \frac{log 3^4}{\log 3} \quad = \quad \frac{4 \log 3}{\log 3} \quad = \quad 4$

(ii) $\log_5 3 \times \log_3 25$

$= \quad \frac{\log 3}{\log 5} \times \frac{\log 25}{3}$

$= \quad \frac{log 25}{\log 5} \quad = \quad \frac{\log 5^2}{\log 3}$

$= \quad \frac{2 \log 5}{\log 5} \quad = \quad 2$

Q5. If log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990, then find the values of the following

(i) log 32 (ii) log 24 (iii) $\log \sqrt{3\frac{1}{3}}$

(iv) log 3 (v) log 30

Solution:

log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990

(i) **log 32** $= \log 2^5 = 5 \log 2$
$= 5(0.3010) = 1.5050$

(ii) **log 24** $= \log 3 \times 8$
$= \log 3 + \log 8$
$= \log 3 + \log 2^3$
$= \log 3 + 3 \log 2$
$= 0.4771 + 3(0.301)$
$= 0.4771 + 0.9030$
$= 1.3801$

(iii) $\log \sqrt{3\frac{1}{3}} = \log \sqrt{\frac{10}{3}}$
$= \log \left(\frac{10}{3}\right)^{1/2}$
$= \frac{1}{2}(\log 10 - \log 3)$
$= \frac{1}{2}(\log 5 \times 2 - \log 3)$
$= \frac{1}{2}[\log 5 + \log 2 - \log 3]$
$= \frac{1}{2}[0.6990 + 0.3010 - 0.4771]$
$= \frac{1}{2}[1.0000 - 0.4771]$
$= \frac{1}{2}(0.5229) = 0.26145$
$= 0.2615$

(iv) $\log \frac{8}{3} = \log 8 - \log 3$
$= \log 2^3 - \log 3$
$= 3 \log 2 - \log 3$
$= 3(0.3010) - (0.4771)$
$= 0.9030 - 0.4771$
$= 0.4259$

(v) **log 30** $= \log 2 \times 3 \times 5$
$= \log 2 + \log 3 + \log 5$
$= 0.3010 + 0.4771 + 0.6990$
$= 1.4771$

EXERCISE 3.4

Q1. Use log tables to find the value of

(i) 0.8176×13.64 **(ii)** $(789.5)^{1/8}$

(iii) $\dfrac{0.678 \times 9.01}{0.0234}$ **(iv)** $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

(v) $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$ **(vi)** $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

(vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$ **(viii)** $\dfrac{(438)^3}{(388)^4}$

Solution:

(i) 0.8176×13.64

Let $x = 0.8176 \times 13.64$

$\log x = \log (0.8176 \times 13.64)$

$= \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1 + 0.1348$

$\log x = 0.9125 + 0.1348$

$\log x = 1.0473$

Taking antilog on both side

Antilog $(\log x) =$ Antilog (1.0473)

$x = 11.15$

(ii) $(789.5)^{1/8}$

Let $x = (789.5)^{1/8}$

$\log x = \log (789.5)^{1/8}$

$= \frac{1}{8} \log 789.5$

$= \frac{1}{8}(2.8974)$

$\log x = 0.3622$

Taking antilog on both side

Antilog $(\log x) =$ Antilog (0.3622)

$x = 2.302$

(iii) $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

$\log x = \log \dfrac{0.678 \times 9.01}{0.0234}$

$= \log 0.678 + \log 9.01 - \log 0.0234$

$$= -1 + 0.8312 + 0.9547 - \bar{2} - 0.3692$$
$$= -1 + 2 + 1.7859 - 0.3692$$
$$= 1 + 1.4167$$
$$\log x = 2.4167$$

Taking antilog on both side

$$\text{Antilog} (\log x) = \text{Antilog} (2.4167)$$
$$x = 261.0$$

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$$x = (2.709)^{1/5} \times (1.239)^{1/7}$$
$$\log x = \log [(2.709)^{1/5} \times (1.239)^{1/7}]$$
$$= \log (2.709)^{1/5} + \log (1.239)^{1/7}$$
$$= \frac{1}{5}(0.4328) + \frac{1}{7} \log 1.239$$
$$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$$
$$= 0.08656 + 0.0133$$
$$= 0.09986$$
$$\log x = 0.0999$$

Taking antilog on both side

$$\text{Antilog} (\log x) = \text{Antilog} (0.999)$$
$$x = 1.258$$

(v) $\frac{(1.23)(0.975)}{(0.0075)(1278)}$

Let $x = \frac{(1.23)(0.975)}{(0.0075)(1278)}$

$$\log x = \log \frac{(1.23)(0.975)}{(0.0075)(1278)}$$
$$= \log[(1.23)(0.6975)] - \log[(0.0075)(1278)]$$
$$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$$
$$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$
$$= -1 + 3 - 3 + 0.9334 - 0.9816$$
$$= -2 + 0.9518$$
$$\log x = \bar{2}.9518$$

Taking antilog on both side

$$\text{Antilog} (\log x) = \text{Antilog} (\bar{2}.9518)$$
$$x = 0.0895$$

(vi) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

$= \left(\dfrac{0.7214 \times 2.37}{60.8}\right)^{\frac{1}{3}}$

$\log x = \log \left(\dfrac{0.7214 \times 2.37}{60.8}\right)^{\frac{1}{3}}$

$= \frac{1}{3}\log \dfrac{0.7214 \times 20.37}{60.8}$

$= \frac{1}{3}\{(\log 0.7214 \times \log 20.37) - \log 60.8\}$

$= \frac{1}{3}\{\log 0.7214 + \log 20.37 - \log 60.8\}$

$= \frac{1}{3}\{\bar{1}.8581 + 1.3090 - 1.7839\}$

$= \frac{1}{3}\{-1 + 0.8581 + 1 + 0.3090 - 1.7839\}$

$= \frac{1}{3}\{1.1671 - 1.7839\}$

$= \frac{1}{3}\{-3 + 3 + 1.1671 - 1.7839\}$

$= \frac{1}{3}\{-3 + 4.1671 - 1.7839\}$

$= \frac{1}{3}\{-3 + 2.3832\}$

$= -1 + 0.7944$

$\log x = \bar{1}.7944$

Taking antilog on both side

Antilog (log x) = Antilog ($\bar{1}.7944$)

$x = 0.6229$

(vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Let $x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$\log x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$= \log\left(83 \times \sqrt[3]{92}\right) - \log\left(127 \times \sqrt[5]{246}\right)$

$= \log 83 + \log(92)^{\frac{1}{3}} - \log 127 - \log(246)^{\frac{1}{5}}$

$= 1.9191 + \frac{1}{3}\log 92 - 2.1038 - \frac{1}{5}\log 246$

$$= 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.3909)$$
$$= 2.5737 - 2.5820$$
$$= -1 + 3.5737 - 2.5820$$
$$= -1 + 0.9917$$
$$\log x = \bar{1}.9917$$

Taking antilog on both side

$$\text{Antilog}(\log x) = \text{Antilog}(\bar{1}.9917)$$
$$x = 0.9811$$

(viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Let $x = \dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$\log x = \log\frac{(438)^3\sqrt{0.056}}{(388)^4}$$
$$= \log(438)^3 + \log\sqrt{0.056} - \log(388)^4$$
$$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$$
$$= 3\log 438 + \frac{1}{2}\log 0.056 - 4\log 388$$
$$= 3\,(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4\,(2.5888)$$
$$= 7.9245 + \bar{1}.3741 - 10.3552$$
$$= 7.9245 - 1 + 0.3741 - 10.3552$$
$$= 8.2986 - 11.3552$$
$$= -4 + 12.2986 - 11.3552$$
$$= -4 + 0.9434$$
$$\log x = \bar{4}.9434$$

Taking antilog on both side

$$\text{Antilog}(\log x) = \text{Antilog}(\bar{4}.9434)$$
$$x = 0.0008778$$

Q2. A gas is expanding according to the law $pV^n = C$. Find C when $p = 80$, $v = 3.1$ and $n = \frac{5}{4}$.

Solution:

$$pv^n = C$$

Substituting $p = 80,\quad v = 3.1,\quad n = \frac{5}{4}$

$$C = 80\,(3.1)^{\frac{5}{4}}$$
$$\log C = \log 80\,(3.1)^{\frac{5}{4}}$$

$$
\begin{aligned}
&= \log 80 + \tfrac{5}{4}\log 3.1 \\
&= 1.9031 + \tfrac{5}{4}(0.4914) \\
&= 1.9031 + \frac{2.570}{4} \\
&= 1.9031 + 0.6143 \\
\log C &= 2.5174
\end{aligned}
$$

Taking antilog on both side

$$
\begin{aligned}
Antilog\,(log\ C) &= \text{Antilog}\,(2.5174) \\
\text{So,}\quad C &= 329.2
\end{aligned}
$$

Q3. The formula $p = 90\,(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution:

$$
\begin{aligned}
p &= 90\,(5)^{-q/10} \\
\log p &= \log\left[90\,(5)^{-\frac{q}{10}}\right] \\
&= \log 90 + \log(5)^{-\frac{q}{10}} \\
&= \log 90 - \frac{q}{10}\log 5 \\
\frac{q}{10}\log 5 &= \log\frac{90}{p}
\end{aligned}
$$

By putting $p = 18$, we have

$$
\begin{aligned}
\frac{q}{10}\log 5 &= \log\frac{90}{18} \\
\frac{q}{10}\log 5 &= \log 5 \\
\frac{q}{10} &= 1 \\
\mathbf{q} &= \mathbf{10}
\end{aligned}
$$

10 units will be demanded.

Q4. If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution:

$$
\begin{aligned}
A &= \pi r^2 \\
\log A &= \log \pi r^2 \\
&= \log \pi + \log r^2 \\
&= \log\frac{22}{7} + 2\log 15 \\
&= \log 22 - \log 7 + 2\log 15
\end{aligned}
$$

$$= 1.342 - 0.8451 + 2(1.1761)$$
$$\log A = 2.8495$$

Taking antilog on both side

$$Antilog\ (\log A) = \text{Antilog}\ (2.8495)$$

So, $A = 707.1$

Q5. If $V = \frac{1}{3}\pi r^2 h$, **find V, when** $\pi = \frac{22}{7}$, $r = 2.5$ **and** $h = 4.2$.

Solution:

$$V = \frac{1}{3}\pi r^2 h$$

By putting the values

$$V = \frac{1}{3} \times \frac{22}{7} \times 25^2 \times 4.2$$
$$V = 22 \times 25^2 \times 0.2$$
$$V = 22 \times 25^2 \times \frac{2}{10}$$
$$\log V = \log\left(22 \times 25^2 \times \frac{2}{10}\right)$$
$$= \log 22 + \log 25^2 + \log\frac{2}{10}$$
$$= \log 22 + 2\log 25 + \log 2 - \log 10$$
$$= 1.3424 + 2.7959 + 0.3010 - 1$$
$$\log V = 4.4393$$

Taking antilog on both side

$$Antilog\ (\log V) = \text{Antilog}\ (4.4393)$$

So, $A = 27.50$

REVIEW EXERCISE 3

Q1. Multiple Choice Questions. Choose the correct answer.

(i) If $a^x = n$, **then..........**

(a) $a = log_x n$ (b) $x = log_n a$

(c) $x = log_a n$ (d) $a = log_n x$

(ii) The relation $y = \log_z x$ **implies**

(a) $x^y = z$ (b) $z^y = x$

(c) $x^z = y$ (d) $y^z = x$

(iii) The logarithm of unity to any base is........

(a) 1 (b) 10

(c) e (d) 0

$$= \quad 1.342 - 0.8451 + 2(1.1761)$$
$$\log A = 2.8495$$

Taking antilog on both side

$$Antilog\ (\log A) = \text{Antilog}\ (2.8495)$$

So, $A = 707.1$

Q5. If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$.

Solution:

$$V = \frac{1}{3}\pi r^2 h$$

By putting the values

$$V = \frac{1}{3} \times \frac{22}{7} \times 25^2 \times 4.2$$
$$V = 22 \times 25^2 \times 0.2$$
$$V = 22 \times 25^2 \times \frac{2}{10}$$
$$\log V = \log\left(22 \times 25^2 \times \frac{2}{10}\right)$$
$$= \log 22 + \log 25^2 + \log\frac{2}{10}$$
$$= \log 22 + 2\log 25 + \log 2 - \log 10$$
$$= 1.3424 + 2.7959 + 0.3010 - 1$$
$$\log V = 4.4393$$

Taking antilog on both side

$$Antilog\ (\log V) = \text{Antilog}\ (4.4393)$$

So, $A = 27.50$

REVIEW EXERCISE 3

Q1. Multiple Choice Questions. Choose the correct answer.

(i) If $a^x = n$, then..........

(a) $a = log_x n$ (b) $x = log_n a$
(c) $x = log_a n$ (d) $a = log_n x$

(ii) The relation $y = \log_z x$ implies

(a) $x^y = z$ (b) $z^y = x$
(c) $x^z = y$ (d) $y^z = x$

(iii) The logarithm of unity to any base is........

(a) 1 (b) 10
(c) e (d) 0

(iv) **The logarithm of any number to itself as base is........**

(a) 1 (b) 0
(c) -1 (d) 10

(v) **$\log e$ =......, where $e \approx 2.718$**

(a) 0 (b) 0.4343
(c) ∞ (d) 1

(vi) **The value of $\log\left(\frac{p}{q}\right)$ is........**

(a) $\log p - \log q$ (b) $\frac{\log p}{\log q}$
(c) $\log p + \log q$ (d) $\log q - \log p$

(vii) **$\log p - \log q$ is same as........**

(a) $\log\left(\frac{q}{p}\right)$ (b) $log(p-q)$
(c) $\frac{\log p}{\log q}$ (d) $\log\left(\frac{p}{q}\right)$

(viii) **$\log(m^n)$ can be written as........**

(a) $(\log m)^n$ (b) $m \log n$
(c) $n \log m$ (d) $\log(mn)$

(ix) **$\log_b a \times \log_c b$ can be written as........**

(a) $\log_a c$ (b) $\log_c a$
(c) $\log_a b$ (d) $\log_b c$

(x) **$\log_y x$ will be equal to........**

(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$
(c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$

Solution:

(i) c	**(ii) b**	**(iii) d**	**(iv) a**	**(v) b**
(vi) a	**(vii) d**	**(viii) c**	**(ix) b**	**(x) c**

Q2. Complete the following.

(i) For common logarithms, the base is......

(ii) The integral part of the common logarithm of a number is called the.......

(iii) The decimal part of the common logarithm of a number is called the......

(iv) If $x = \log y$, then y is called the..........of x.

(v) f the characteristic of the logarithm of a number is $\bar{2}$, that number will have.............zero(s) immediately after the decimal point.

(vi) If the characteristic of the logarithm of a number is 1, that number will have..........digits in its integral part.

Answers:

(i)	**10**	**(ii)**	**Characteristic**
(iii)	**Mantissa**	**(iv)**	**Antilogarithm**
(v)	**One**	**(vi)**	**2**

Q3. Find the value of x in the following.

(i) $\log_3 x = 5$ **(ii)** $\log_4 256 = x$

(iii) $\log_{625} 5 = \frac{1}{4} x$ **(iv)** $\log_{64} x = \frac{-2}{3}$

Solution:

(i) $\log_3 x = 5$

$3^5 = x$

$x = 3^5$

So, $x = 243$

(ii) $\log_4 256 = x$

$4^x = 256$

$4^x = 4^4$

So, $x = 4$

(iii) $\log_{625} 5 = \frac{1}{4} x$

$(625)^{\frac{1}{4x}} = 5$

$(5^4)^{\frac{1}{4x}} = 5$

$5^x = 5^1$

So, $x = 1$

(iv) $\log_{64} x = \frac{-2}{3}$

$(64)^{\frac{-2}{3}} = x$

$(4^3)^{\frac{-2}{3}} = x$

$4^{-2} = x$

$x = \frac{1}{4^2}$

So, $x = \frac{1}{16}$

Q4. Find the value of x in the following.

(i) $\log x = 2.4543$ **(ii)** $\log x = 0.1821$

(iii) $\log x = 0.0044$ **(iv)** $\log x = \bar{1}.6238$

Solution:

(i) $\log x = 2.4543$

From the table against the row of 0.45 under 4 we have 2844 and difference under 3 is 2. Adding 2844 and 2 we get 2846.

So $x = 284.6$

(ii) $\log x = 0.1821$

From the table against the row of 0.18 under 2 we have 1521 and difference under 1 is 0. Adding 1521 and 0 we get 1521.

So $x = 1.521$

(iii) $\log x = 0.0044$

From the table against the row of 0.00 under 4 we have 1009 and difference under 4 is 1. Adding 1009 and 1 we get 1010.

So $x = 1.010$

(iv) $\log x = \bar{1}.6238$

From the table against the row of 0.62 under 3 we have 4198 and difference under 8 is 8. Adding 4192 and 8 we get 4206.

So $x = 0.04206$

Q5. If log 2 = 0.3010, log 3 = 0.4771 and log 5= 0.6990, then find the values of the following.

(i) $\log 45$ **(ii)** $\log \frac{16}{15}$ **(iii)** $\log 0.048$

Solution:

(i) $\log 45$

$= \log 3 \times 3 \times 5$

$= \log 3 + \log 3 + \log 5$

$= 0.4771 + 0.4771 + 0.6990$

$= 1.6532$

(ii) $\log \frac{16}{15}$

$= \log 16 - \log 15$

$= \log 2^4 - \log 3 \times 5$

$= 4\log 2 - \log 3 - \log 5$
$= 1.2040 - 1.1761$
$= 0.0279$

(iii) $\log 0.048$

$= \log \frac{48}{1000}$
$= \log 48 - \log 1000$
$= \log 3 \times 16 - \log 10^3$
$= \log 3 + \log 16 - \log 10^3$
$= \log 3 + \log 16 - 3\log 10$
$= 0.4771 + \log 2^4 - 3(1)$ $\qquad (\because \log 10 = 1)$
$= 0.4771 + 4\log 2 - 3$
$= 0.4771 + 4(0.3010) - 3$
$= 0.4771 + 1.2040 - 3$
$= 1.6811 - 3$
$= -2 + 0.6811$
$= \bar{2}.6811$

Q6. Simplify the following.

(i) $\sqrt[3]{25.47}$ **(ii)** $\sqrt[5]{342.2}$

(iii) $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Solution:

(i) $\sqrt[3]{25.47}$

Let $x = \sqrt[3]{25.47} = (25.47)^{\frac{1}{3}}$

$\log x = \log(25.47)^{\frac{1}{3}} = \frac{1}{3}\log 25.47$

$= \frac{1}{3}(1.4060)$

$\log x = 0.4686$

Taking antilog on both side

$Antilog(\log x) =$ Antilog (0.4686)

So, $x = 2.942$

(ii) $\sqrt[5]{342.2}$

Let $x = \sqrt[5]{342.2} = (342.2)^{\frac{1}{5}}$

$\log x = \log(342.2)^{\frac{1}{5}} = \frac{1}{5}\log 342.2$

$= \frac{1}{5}(2.5343)$

$\log x = 0.5069$

Taking antilog on both side

$$Antilog\ (log\ x) = \text{Antilog}\ (0.5069)$$

So, $x = 3.213$

(iii) $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Let $x = \dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

$$\log x = \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$$

$$= \log\ (8.97)^3 \times (3.95)^2 - \log\ (15.37)^{\frac{1}{3}}$$

$$= \log(8.97)^3 \times log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$$

$$= 3 \log\ 8.97 + 2 \log\ 3.95 - \frac{1}{3} \log 15.37$$

$$= 3\ (0.9528) + 2\ (0.5966) - \frac{1}{3}\ (1.1867)$$

$$= 2.8584 + 1.1932 - 0.3956$$

$$= 4.0516 - 0.3956$$

$$\log x = 3.6560$$

Taking antilog on both side

$$Antilog\ (log\ x) = \text{Antilog}\ (3.6560)$$

So, $x = 4529$